MACROECONOMIC MODEL FOR POLICY ANALYSIS AND INSIGHT
(A DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODEL
FOR THE BANGKO SENTRAL NG PILIPINAS)

by


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Macroeconomic Model for Policy Analysis and Insight (a Dynamic Stochastic General Equilibrium Model for the Bangko Sentral ng Pilipinas)


Abstract

A DSGE model is dynamic in the sense that it explains how the economy evolves over time. It is stochastic because agents know only the distribution of future shocks. Thus, only when these models are linearized to the first order do agents behave as if future shocks are equal to zero, which is the certainty equivalence property. Finally, it is based on a general equilibrium framework as it depicts the macroeconomy as the sum of individual choices and decisions made by firms, households, the government, and the central bank, according to their own preferences and views about the future.

This paper presents the initial specification of and results of the BSP's DSGE model for the Philippine economy. The development of the model complements existing models used by the BSP for policy simulation. The basic model starts at the level of individuals and firms who are assumed to make rational decisions on how much to save, spend or invest based on their preferences and available choices. In this approach, the macroeconomy is seen as the sum of individual choices and decisions (the microeconomy). This basic framework is extended to an open economy setting and embeds price and financial rigidities.

The paper employs Bayesian estimation, which works best with state of the art theoretical models. In this method, one can formulate prior distributions about the structural parameters and not about reduced form coefficients. Posterior distributions of the parameters can be obtained by calculating the likelihood functions based on observed data. Since the method works with likelihood functions, it actually goes beyond matching selected moments of artificially-generated data with actual data. With the advances in approximation methods, random number generators and sampling method like the Monte Carlo Markov Chain (MCMC), there is no need to restrict the parameter distribution to normal distributions.

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1. Introduction

Traditionally, the empirical analysis of economic phenomena is carried out  
through the use of econometric models, which can represent many fundamental  
economic relationships, depending on the purpose of the modeler. However,  
econometric models are also susceptible to ad hoc specifications that improve their fit  
with actual data but do not necessarily conform with economic theory. Moreover,  
changes in government policy can invalidate predictions from econometric models by  
altering economic relationships and the behavior of households and firms.

Dynamic stochastic general equilibrium (DSGE) models have evolved over the  
years in response to the limitations of structural macroeconomic models. The basic  
principle of DSGE macroeconomic modeling is to start at the level of individuals and  
firms who are assumed to make rational decisions on how much to save, spend or invest  
based on their preferences and available choices. In this approach, the macroeconomy  
is seen as the sum of individual choices and decisions (the microeconomy). The  
availability of powerful computer hardware and software for general use has made  
possible the computation of these micro-founded mathematical models.

A DSGE model is dynamic in the sense that it explains how the economy  
evolves over time. It is stochastic because agents know only the distribution of future  
shocks. Thus, only when these models are linearized to the first order do agents behave  
as if future shocks are equal to zero, which is the certainty equivalence property.  
Finally, it is based on a general equilibrium framework as it depicts the macroeconomy  
as the sum of individual choices and decisions made by firms, households, the  
government, and the central bank, according to their own preferences and views about  
the future.

1.1 Motivation

The development of a DSGE model for the Bangko Sentral ng Pilipinas (BSP)  
aims to complement rather than replace existing models used by the BSP. Despite the  
criticisms on econometric models, they still maintain their usefulness. Just like other  
models, DSGE models cannot exhaustively analyze every single policy issue confronting  
the central bank. Even a simple extension of the model to examine a specific policy issue  
could pose complications for other equations of the model, a task that is not easy even  
for a seasoned modeler. Building a DSGE model is not only time-intensive but also  
demanding in terms of resources and the required technical skills. Moreover, learning the

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nuances of DSGE modeling takes longer because the training and experience of staff
tend to relate more to traditional structural macro models and vector autoregressive
(VAR) models and not to micro-founded models.

In this paper, we capture key characteristics of the Philippine economy using the
DSGE framework. The microfoundations of DSGE models make them more appealing
for policy evaluation because the relationships embodied do not change with changes in
the policy environment. Characteristically, DSGE models can work with limited data
sources and make use of prior information (i.e., previous studies, expert judgment and
prior beliefs) in calibrating the parameters of the model.

We then put the model to work, to see what its implications are with respect to the
responses of key variables to internal and external shocks, and the way alternative policy
regimes can change how key variables respond to such internal and external changes.
**The goal is insight.** Once we have a dynamic general equilibrium model in place,
however approximate, analysts and policymakers are forced to think in terms of the
intertemporal constraints on their actions. Changes in debt, for example, will generate
increases in risk premia, which will have effects on the exchange rate.

### 1.2 The Debate on DSGE Modeling

Many policy makers and applied researchers, especially those trained in classical
econometrics, have reservations and implicit qualms about DSGE modeling and
calibration exercises. The oft-cited criticism is that researchers are delving into "black
box" because the behind-the-scenes workings are too complicated to be understood by
researchers. Furthermore, others contend that since DSGE models are not estimated
with actual data, they are nothing more than sophisticated computer games for
economists.

The typical Minnesota-style response to these criticisms, often enunciated by V.K
Chari, is that we should look for progress, and therefore should never "regress", literally
and figuratively. Regressions can be misleading about what is going on in the economy.
In regression, we have to work, most if not all of the time, with reduced form models,
which tell us little or nothing about causality and deeper economic relationships. Chari
has shown this very powerfully with examples of artificial data generated by models with
underlying productivity shocks and passive monetary policy. In the underlying data
generating mechanism, the forcing variable is productivity. However, when he used the
artificial data for estimation of reduced form equations, he could not reject the hypothesis
that monetary shocks were the true forcing variables.

The Minnesota attack on classical econometric estimation, at least in
macroeconomics, is reminiscent of the identification problem that dates back to the work
of Koopmans, Hood and Marshak at the Cowles Commission for Economic Research.
For example, if we regress income growth on educational expenditures, are we finding
out if educational investment is a cause of income growth, or vice versa, that income
growth is a cause of educational investment, or simply that both are correlated,
responding in tandem to an unspecified third factor. Due to this identification problem,
meaningful empirical work requires formulation of a theoretical model with appropriate
restrictions on the coefficients of the estimating equations.

The rational expectations critique of econometric models by Robert Lucas and
the DSGE revolution that began with the real business cycle theory of Kydland and
Prescott pushed the identification critique of empirical estimation to its limit. Parameters
of empirically estimated models are not structural parameters or deep parameters. They
depend on policy regimes. Thus, coefficients of a consumption function, estimated under
one tax regime, will not be the same under a new tax regime. This puts into question the policy usefulness of such econometrically estimated models.

The attack on econometric models led to the calibration method. With this approach, the values of key structural parameters of the model may be obtained from cross section studies, informed judgment or own prior beliefs. Then we solve and simulate the model, and see how well the properties of the model-generated data match properties of actual data. This is an example of indirect inference.

The calibration approach is honest about what we are able and not able to do with our models. At the very least it puts up front for the policy analyst that, in the end, we have to work with our models, we have to draw on what economic theory has to offer. We are not soothsayers. We have to go back to our models for insight into real world data.

In the heydays of the large models, there was often a disjuncture between macroeconomic theory and empirical macro. For example, elaborate theoretical models of investment dynamics with adjustment costs are studied, but when it came time to empirical work, investment function is simply estimated with simple lag structures that have nothing to do with the models of investment behavior (for example, lags with geometrically declining weights). So the Minnesota attack on empirical models, for all of its iconoclasm, has had the effect of bringing together more closely good dynamic economic theory with good dynamic econometrics, when we match moments of data generated by theoretical models with subsets of actual data. Put another way, if we have to use models in empirical work, why not work with the models that our theory has to offer?

The rational expectations and real business cycle (RE/RBC) attack on econometric models, championed in large part by the Minnesota iconoclasm of V.K Chari and his colleagues Patrick Kehoe and Ellen McGratten of the Minneapolis Federal Reserve Bank, powerful as it is, in the end, may have gone too far. It is nothing less than econometric nihilism. Is there nothing more that we can do, with all the tools of empirical science, other than simply match moments, between artificially generated data from state-of-the-art models and actual data?

This is where Bayesian macroeconomics comes to the fore and is the estimation method used in this paper. In the econometric method, parameters, which are assumed to be fixed across time, are estimated. The Bayesian approach, on the other hand, does not make such an assumption. In fact, parameters can change and they even have their own distributions. Thus, we can formulate prior distributions about the underlying structural parameters of the model, not about reduced form coefficients. We can then obtain the posterior distributions of these parameters once we calculate the likelihood functions, based on observed data. Since we are working with the likelihood function, we are going beyond matching selected moments of artificially-generated data with actual data.¹

Advanced computational methods have enabled researchers to take the Bayesian approach to a powerful new level and usefulness in policy research and evaluation. Earlier works of Zellner show how posterior distributions of parameters can be derived analytically from the theoretical distributions and the analytical form of the

¹ As Hogg, McKean, and Craig remind us in their sixth edition of Introduction to Mathematical Statistics, in drawing inference about any parameter \( \theta \), after \( X \) are observed, that is, “all relevant information is contained in the likelihood function”.

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likelihood function. Since he wished to obtain closed form solutions for the posterior distributions, Zellner had to restrict himself to normal prior distributions. In this case, he was able to work with conjugate priors: posterior distributions are transformations of the prior distributions, with the same functional form. With modern numerical analysis, specifically advances in approximation methods, random number generators, and sampling methods such as the Monte Carlo Markov Chain (MCMC), there is no need to restrict ourselves to normally-distributed priors about the structural parameters of the model.

2. The Framework of the BSP's DSGE Model

The theoretical framework is that of a small open economy. In a nutshell, the stylized model of the economy consists of various interacting sectors, captured by the following flow chart in Figure 1.

![Figure 1. Schematic Diagram of the BSP-DSGE Model for the Philippine Economy](image)

Households supply labor to firms, consume goods and make deposits in banks. Firms produce goods for domestic consumption and export and borrow for working capital from banks. Banks accept deposits from households, lend to the government and firms, and borrow from foreign creditors and financial centers. The government sector takes in taxes (based on labor income as well as consumption) and provides goods to the households. The central bank sets the policy rate, which affects the return on government bonds. The return on government bonds, in turn, affects the lending and deposit rates.

Foreign demand is assumed to depend in part on exogenous world developments and in part on the exchange rate. Other sources of change in the model come from productivity, terms of trade, foreign interest rates, risk premia charged on the interest rates, and shocks to the financial sector.

Each set of "players", such as households, firms, banks, the central bank, international financial centers, the fiscal authority, and foreigners demanding exports goods, comes with specific sources of risks for the economy. The shocks or sources of risk in the model are shown in circles in Figure 2.
Households carry the risk of changes in their rate of time preference for present and future consumption. Firms face risk in the form of changes in terms of trade and productivity and well as varying degrees of markup pricing due to monopolistic competition. Demanders of export goods, of course, face risk of global upturns and downturns, which affect their demand for goods from emerging markets. The fiscal authority faces pressure for increased spending in boom times, thereby accelerating demand and inflationary pressures. International financial centers in turn face risks of bullish and bearish behavior in emerging markets, and in particular are vulnerable to regional contagion. Banks face risks of default of their loans, and increases or decreases in the costs of financial intermediation. Finally, the central bank is faced with the risk of having to adjust policy because of unanticipated shocks.

Figure 2 further implies that risks to one set of players quickly spill over to other sets of players. For example, an increase in the risk premium demanded by international foreign centers leads to higher interest rates, which reduce lending to firms or put pressures for further liquidity support from the central bank, leading to inflationary pressures. Pressures for pro-cyclical government spending of course, leads to more borrowing from banks, and more borrowing by banks from foreign financial centers, which in turn can lead to increasing risk premia on foreign loans.

2.1 The Brass Tacks of the Framework

Following Thomas J. Sargent's approach to macroeconomics, we begin by specifying preferences and endowments for households, and technology for firms, as well as the objectives, constraints, risks, and reaction functions of the fiscal authority, the central bank, private banks, international financial centers, foreign consumers -- the strategy of the players in the model. Once these components are specified, we have nailed down the "laws of motion" of the model for the key variables of the model, specifically overall inflation but also economic growth and financial stability.
2.1.1 Household Preferences and Endowments

Households own capital, for rental to export firms, and supply labor both to these firms and the home-goods producing firms. The following law of motion is specified for capital, while adjustment costs are given by $AC_t$.

$$K_t^x = (1 - \delta)K_{t-1}^x + I_t^x$$

$$AC_t = \left( \frac{\phi(I_t^x - \delta K_t^x)}{2K_t} \right)^2$$

Capital for rental to export firms ($K_t^x$) depreciates at the rate $\delta$. When households accumulate capital or decumulate capital beyond the steady state level, they pay adjustment costs. The parameter $\phi$ is the adjustment cost parameter. We further assume that the investment goods $I_t^x$ are imported from abroad, and that the price $P_t^f$ is the relevant price for these goods. The variable $\bar{K}^x$ is the steady state level of the capital stock.

Household consumption (at time $t$), $C_t$, follows the constant elasticity of substitution (CES) function consisting of bundle of both domestic consumption goods, $C_t^d$ and imported goods, $C_t^f$.

$$C_t = \frac{\theta_1^{-\frac{1}{\theta_1}}}{(1 - \lambda_1)^{\frac{1}{\theta_1}}} \left( C_t^d \right)^{\theta_1} \frac{\theta_1^{-\frac{1}{\theta_1}}}{(\lambda_1)^{\frac{1}{\theta_1}}} \left( C_t^f \right)^{\theta_1}$$

The demand for each component of consumption is a function of the overall consumption index and the price of the respective component relative to the general price level, $P_t$. The parameters $\lambda_1$ and $(1 - \lambda_1)$ are the relative shares of foreign and domestic goods in the overall consumption index, while $\theta_1$ is the price elasticity of demand for each consumption component.

$$C_t^d = (1 - \lambda_1) \left( \frac{P_t^d}{P_t} \right)^{\theta_1} C_t$$

and

$$C_t^f = \lambda_1 \left( \frac{P_t^f}{P_t} \right)^{\theta_1} C_t$$

Domestically-produced goods are both non-traded home goods and export goods (some of which are consumed domestically). The following CES aggregator is used for domestically-produced consumption goods:

$$C_t^d = \left( 1 - \lambda_2 \right) \lambda_2 \left( C_t^h \right)^{\frac{1}{\theta_2}} + \lambda_2 \left( C_t^x \right)^{\frac{1}{\theta_2}}$$

The relative demands for the home non-traded goods and the export goods are given by the following equations where the parameters $\lambda_2$ and $(1 - \lambda_2)$ are the shares of the export and non-traded goods in domestic production of consumption goods, and $\theta_2$ is the price elasticity of demand.
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\[ C^h_t = (1 - \lambda_2) \left( \frac{P^h_t}{P_t} \right)^{-\theta_2} C^d_t \quad \text{and} \quad C^s_t = \lambda_2 \left( \frac{P^s_t}{P_t} \right)^{-\theta_2} C^d_t \]

The domestically-produced price index is given by the following CES aggregator:

\[ P^d_t = \left[ (1 - \lambda_2)(P^h_t)^{-\theta} + \lambda_2 (P^s_t)^{-\theta} \right]^{-\frac{1}{\theta}} \]

In the same manner, the overall price index, of course, is a CES function of the price of foreign and domestic consumption goods:

\[ P_t = \left[ (1 - \lambda_1)(P^d_t)^{-\theta} + \lambda_1 (P^f_t)^{-\theta} \right]^{-\frac{1}{\theta}} \]

The following equation gives the household budget constraint. Households earn wage income \( (W_t, L_t) \), interest income from previous period’s deposits \( (1 + R_{t-1}^d)M_{t-1} \), return on productive capital rented to the export firm \( (P_t^k, K_t^e) \) and dividends from the export and non-traded or home-goods producing firms \( (\Pi_t = \Pi_t^e + \Pi_t^n) \). In addition to buying consumption goods at consumption tax \( \tau_c \), households put deposits \( M_t \) in the bank and pay taxes on labor income \( (\tau_l) \) and face adjustment cost when they increase or decrease capital beyond its steady state level.

\[ W_t L_t + (1 + R_{t-1}^d)M_{t-1} + P_t^k K_t^e = P_t C_t (1 + \tau_c) + M_t + \tau W_t L_t + P_t^f I_t^s + P_t^f \left( \frac{\phi(I_t^f - \delta K_t^s)}{2 K_t} \right) \]

We assume that government spending \( G \) is bundled with consumption for utility in CES aggregator as the former creates externalities for consumption in the form of public utilities, infrastructure and other services that enhance household utility:

\[ \tilde{C}_t = \left[ \mu C_t^{-\eta} + (1 - \mu) G_{t-1}^{-\eta} \right]^{-\frac{1}{\eta}} \]

However, household utility does not simply come from the current consumption bundle. Rather, habit persistence applies to this consumption index when it enters the utility function, so that the relevant consumption index is deflated by the habit stock, \( H_t \). The habit stock is a function of the lagged average consumption bundle, raised to the power \( \rho_h \) or the habit persistence parameter:

\[ H_t = \tilde{C}_t^{\rho_h} \]

Overall utility is a positive function of the consumption bundle and the habit stock and a negative function of labor. The parameter \( \sigma \) is the relative risk aversion coefficient while \( \gamma \) is the disutility of labor and \( \sigma \gamma \) is the Frisch labor supply elasticity.
The household chooses the paths of consumption, labor, deposits, investment and capital, to maximize the present value of its utility function subject to the budget constraint and the law of motion for capital. Thus, the objective function of the household is given by the following expression, where the parameter $\beta$ represents the constant, exogenous discount factor.

$$\max_{C_t, L_t, I_t, K_t} \sum_{i=0}^{\infty} \beta U(\tilde{C}_{t+i} / H_{t+i}, L_{t+i})$$

This optimization is subject to the two constraints:

1. $W_t L_t + (1 + R_t^d)M_{t-1} + \Pi_t + P_t \epsilon K_t^x = P_t C_t (1 + \tau_c) + M_t + \tau W_t L_t + P_t^I I_t^x + P_t^f \left[ \frac{\phi(I_t^f - \delta K_t^x)}{2K_t} \right]$  
2. $K_t^x = (1 - \delta)K_{t-1}^x + I_t^x$

The household optimization can be more compactly represented by the following recursive Bellman equation:

$$V_t = \max_{C_t, L_t, I_t, K_t} \sum_{i=0}^{\infty} \beta^i \{ -\Lambda_t U(\tilde{C}_t / H_t, L_t, P_t C_t (1 + \tau_c) + M_t - (1 + R_t^d)M_{t-1} + W_t L_t - \Pi_t - P_t^I K_t^x - Q_t^x (K_t^x - I_t^x - (1 - \delta)K_{t-1}^x) \}$$

Note that there are two Lagrange multipliers: $\Lambda_t$ is the familiar marginal utility of income or wealth, while $Q_t$, known as Tobin’s Q, is the shadow price of capital.

Optimizing the Bellman equation with respect to the decision variables $C_t, L_t, M_t, I_t, K_t$ yields the following set of Euler or First-Order Conditions (FOCs) for the representative household:

- **FOC1**: $\frac{\partial V_t}{\partial C_t} = \Lambda_t P_t \left[ \frac{\tilde{C}_t / H_t}{\beta} \right] = \frac{1}{H_t} (C_t)^{1-\eta} \mu(C_t)^{-\eta-1}$
- **FOC2**: $\frac{\partial V_t}{\partial L_t} \Rightarrow \gamma L_t = \Lambda_t (1 - \tau)W_t$
- **FOC3**: $\frac{\partial V_t}{\partial M_t} \Rightarrow \Lambda_t = \beta \Lambda_{t-1} (1 + R_t^d)$
- **FOC4**: $\frac{\partial V_t}{\partial K_t} \Rightarrow Q_t^x = \beta \Lambda_{t-1} P_t^I + \beta \Lambda_{t-1} P_t^m \left[ \frac{\phi(I_{t-1}^x - \delta K_{t-1}^x)}{2(K_{t-1}^x)^2} \right] + \beta Q_{t-1}^x (1 - \delta)$
- **FOC5**: $\frac{\partial V_t}{\partial I_t} \Rightarrow I_t^x = N_t K_t^x + K_t^x \left[ \frac{Q_t^x}{\Lambda_t} - P_t^m \right]$
The first equation, FOC1, simply tells us that the marginal utility of wealth is equal to the marginal utility of consumption divided by the price level. The second equation, FOC2, states that the marginal disutility of labor is equal to the after-tax marginal utility of consumption allowed by the after-tax wage. The third equation, FOC3, is the Keynes-Ramsey rule for optimal saving: the marginal utility of wealth today should be equal to the discounted marginal utility tomorrow, multiplied by the gross rate of return on saving (in the form of deposits).

The equation for Tobin’s Q, given by FOC4, tells us that the value of capital today is the discounted marginal utility of capital tomorrow, multiplied by the return to capital, in addition to the reduced value of adjustment costs in the future (due to the higher level of capital) and the discounted value of capital tomorrow, net of depreciation.

Finally, the investment equation, FOC5, tells us that investment will be equal to the steady state investment, $\delta K_{ss}^x$, when $\frac{Q_t^f}{A_t} = P_t^f$. Any increase in Tobin’s Q, relative to the marginal utility of income and the price of investment goods, will trigger increases in investment.

### 2.1.2 Production and Technology

The model has three types of firms: the export firms $Y_t^x$, the home goods producers $Y_t^h$, and importers $Y_t^f$:

$$Y_t = Y_t^x + Y_t^h + Y_t^f$$

(a) **Export Firms**

The export firms produce their goods with the following CES technology:

$$Y_t^x = Z_t^x A_t^x \left[ (1 - \alpha_t) (L_t^x)^{\kappa_t} + \alpha_t (K_t^x)^{\kappa_t} \right]^{-\frac{1}{\kappa_t}}$$

The parameters $\alpha_t$ and $(1 - \alpha_t)$ are the shares of capital and labor in the export production function, respectively. The coefficient $\kappa_t$ is the CES aggregator. The technology shock is given by $Z_t^x$. We assume that this technology shock evolves according to the following stochastic process:

$$\ln(Z_t^x) = \rho_t \ln(Z_{t-1}^x) + (1 - \rho_t) \ln(\bar{Z}^x) + \epsilon_{Z_t^x,t}^x; \quad \epsilon_{Z_t^x,t}^x \sim N(0, \sigma_{\epsilon_{Z_t^x,t}}^2),$$

where $\bar{Z}^x$ is the steady state value of the shock and $\rho_t$ is the autoregressive parameter.

The demand for the export good can be both for domestic consumption, as well from foreign demand: $Y_t^x = C_t^x + C_t^f$. We assume that foreign demand responds to the relative price of this export good, in the sense that if the real exchange rate depreciates relative to the steady-state level $\left( \frac{\bar{S}}{P} \right)$, foreign demand rises by a factor $\chi_t^x$. 

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\[
\ln(C^t_i) = \ln(C^t_o) + \chi \left[ \ln\left( \frac{S_{t+1}}{P_{t+1}} \right) - \ln\left( \frac{\bar{S}_t}{\bar{P}_t} \right) \right]
\]

We assume that the firm faces a liquidity constraint, it must borrow an amount \(N^x_t\) from banks each quarter to pay a fraction of its wage bill, at the borrowing rate \(R^x_t\). We also assume that the amount of borrowing is subject to a collateral constraint proportional by a factor to the total returns on capital:

\[
N^x_t = \phi W^x_t L^x_t
\]

\[
N^x_t \leq \nu_1 P^x_t K^x_t
\]

The total profits (or dividends) of the export firm is given by the following identity:

\[
\Pi^x = P^x_t Y^x_t - (1 + \phi R^x_t) W^x_t L^x_t - P^x_t Y^x_t
\]

Maximizing profits with respect to the use of capital and labor, we have the following first-order conditions for the firm:

FOC 1:

\[
\frac{\partial Y^x_t}{\partial L^x_t} = (1 + \phi R^x_t) \frac{W^x_t}{P^x_t}
\]

FOC 2:

\[
\frac{\partial Y^x_t}{\partial K^x_t} = \frac{P^k_t}{P^x_t}
\]

Under a small open economy setting, we assume that the price of the export good in domestic currency is simply equal to the exchange rate \(S_t\) multiplied by the world export price, \(P^x_t\). We assume that the world export price follows the following exogenous stochastic process:

\[
\ln(P^x_t) = \rho_{\delta \varepsilon} \ln(P^x_{t-1}) + (1 - \rho_{\delta \varepsilon}) \ln(\bar{P}^x_t) + \varepsilon_{\delta \varepsilon, t} \sim N(0, \sigma_{\delta \varepsilon}^2)
\]

(b) Home Goods

The firm producing home goods faces a simple production function, with a fixed unitary stock of capital: \(Y^h_t = Z^h_t (L^h_t)^{-\alpha_t}\). The technology shock to home-good production follows a similar process as the export-technology shock:

\[
\ln(Z^h_t) = \rho_{\varepsilon \varepsilon} \ln(Z^h_{t-1}) + (1 - \rho_{\varepsilon \varepsilon}) \ln(\bar{Z}) + \varepsilon_{\varepsilon \varepsilon, t} \sim N(0, \sigma_{\varepsilon \varepsilon}^2)
\]

\[\text{2}^2\text{ In the CES technology, we have the following expressions: } \frac{\partial Y^x_t}{\partial L^x_t} = (A^x Z^x_t)^\alpha (1 - \alpha_t) \left( \frac{\bar{Y}^x_t}{L^x_t} \right)^{1-\alpha_t}\text{ and } \frac{\partial Y^x_t}{\partial K^x_t} = (A^x Z^x_t)^\alpha (\alpha_t) \left( \frac{\bar{Y}^x_t}{L^x_t} \right)^{1-\alpha_t}\text{. With } \kappa_1 = 0, \text{ the first order conditions reduce to the Cobb-Douglas marginal productivity conditions.}\]
These firms also facing a liquidity constraint for meeting their wage bill: 
\[ N_t^h = \phi_s W_t L_t^h \]. The profits of the home-goods firms are given by the following relation: 
\[ \Pi_t^h = P_t^h Y_t^h - (1 + \phi_s) R_t^s W_t L_t^h \]. Optimizing profits implies the following first-order condition for 
cost minimization:

\[
\frac{\partial Y_t^h}{\partial L_t^h} = (1 + \phi_s) \frac{W_t}{P_t^h}
\]

**Calvo Pricing for Home Goods**

Unlike the export firms, the pricing for home-goods firms is governed by the 
behavior of sticky monopolistically competitive firms. Let the marginal cost at time \( t \) be 
given by the following expression:

\[
MC_t = \frac{(1 + \phi_s) W_t}{(1 - \alpha_z) Y_t^h / L_t^h}
\]

In the Calvo price setting world, there are forward-looking price setters \( P_t^o \) and 
backward looking setters \( P_t^{h,b} \).

\[
P_t^h = \left[ \xi (P_t^{h,b})^{1-\xi} + (1-\xi)(P_t^o)^{1-\xi} \right]^{\frac{1}{1-\xi}}
\]

Assuming at time \( t \), a probability of persistence of the price at \( \xi \), with demand 
for the product from firm \( i \) given by \( Y_t^i(P_t^h)^{\gamma^i} \), the expected marginal cost, in recursive 
formulation, is presented by the expression for \( MC_t^{num} \). The expected demand, for the 
given price, is given by the variable \( MC_t^{den} \).

\[
MC_t^{num} = Y_t^h P_t^h^{\gamma^h} MC_t + \beta \xi A_{t+1}^{num} \quad \text{and} \quad MC_t^{den} = Y_t^h (P_t^h)^{\gamma^h} \beta \xi A_{t+1}^{num}
\]

The forward-looking price setting sets the optimal price, \( P_t^o \), so that expected 
marginal revenue is equal to expected marginal costs, \( P_t^o = \frac{MC_t^{num}}{MC_t^{den}} \).

The backward looking price setters do not keep the price fixed. They will set their 
price equal to the price at the previous period, \( P_{t-1}^b \) multiplied by the previous period's 
inflation, \( P_{t-1}^b / P_{t-2} \) raised to an indexation parameter \( \kappa^d \), and by the gross inflation target 
announced by the central bank, \( (1 + \pi_t^d) \), which represents monetary policy statements 
relative to inflation targets, raised to a parameter \( \kappa^c \).

\[
P_t^{h,b} = P_{t-1}^b \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\kappa^d} (1 + \pi_t^d)^{\kappa^c}
\]
(c) Importing Firms

Imported goods $Y^f$ are used for both consumption $C^f$ and for investment in the export sector $I^x: Y^f = C^f + I^x$. The importing firms do not produce these goods. However, they have to borrow a fraction of the cost of these imported goods in order to bring them to the home market for domestic consumers and investors:

$$N_i^f = \phi_3(S_tP'^w_tY^f_t)$$

where $P'^w_t$ is the world price of the import goods and $S_t$ is the exchange rate. The domestic cost of the imported goods is given by:

$$P^w_t = \left[ \phi_1 \left( 1 + R_t^d \right) - (1 - \phi_2) \right] S_t P'^w_t$$

$$[1 + \phi_1 R_t^s]S_t P'^w_t$$

2.1.3 The Financial Sector

Banks lend to all three types of firms: $N_t = N^d_t + N^b_t + N^f_t$. In addition to these firms, the banks lend to the government $B^g_t$ and receive a risk-free interest rate $R_t$. They borrow from foreign financial centers the amount $B^f_t$ and pay a risk premium above the domestic interest rate when such foreign debt exceeds a steady-state level $\Phi_f$:

$$\Phi_f = \max \left\{ 0, \phi_3 \left[ e^{\left| R^d_t - R^f_t \right|} - 1 \right] B^f_{t-1} \right\}$$

The banks thus pay a gross interest rate $R^f_t + \Phi_f$ on their outstanding dollar-denominated debt $B^f_t$ to foreign financial centers.

In addition to paying the interest rate $R^d_t$ to deposits, we assume that banks are also required to set aside a required ratio of reserves on outstanding deposits, $\phi_4 M_t$. The relevant opportunity cost of holding these reserves is of course the amount the banks can earn by holding risk-free government bonds, $\phi_4 R_t^d M_t$. In addition, banks are required to set aside a fraction of capital against their outstanding loans, $\phi_5 N_t$. As in the case of the required reserves against deposits, the opportunity cost is given by $\phi_5 R_t^d N_t$.

The gross profit of the banking sector is given by the following balance-sheet identity:

$$\Pi^b_t = (1 + R^d_{t-1})B^g_{t-1} + (1 + R^d_{t-1})N^d_{t-1} + S_t B^f_t + M_t - (1 + R^d_{t-1})M^d_{t-1} - \phi_t B^g_{t-1} N^d_{t-1} - \phi_4 R^d_{t-1} M^d_{t-1} - B^f_t - N_t - \phi_5 R^d_{t-1} N_t$$

The bank maximizes the present discounted value of its profits, given by $V^b_t$, with respect to its portfolio of assets (loans to the government and firms, $B^g_t$ and $N_t$) and liabilities (deposits from households and borrowing from foreign financial centers.
This optimization leads to the following set of first-order conditions for financial sector profit maximization:

$$\frac{\partial V_t}{\partial B_t^f} = \beta (1 + R_t^f)$$

$$\frac{\partial V_t}{\partial M_t} = \beta (1 + R_t^f) + \beta \phi_b R_t^d$$

$$\frac{\partial V_t}{\partial N_t} = \beta (1 + R_t^f) - \beta \phi_b R_t^d$$

$$\frac{\partial V_t}{\partial B_t^f} = S_t = \beta (1 + R_t^f + \Phi_t) S_{t+1} + (1 + R_{t-1} + \Phi_{t-1}) B_{t-1}^f$$

This set of first-order conditions leads to the familiar set of spreads for interest rates, as well as the interest-parity equation:

$$\beta = \frac{1}{1 + R_t^b}$$

$$R_t^b = (1 + \phi_b) R_t^d$$

$$R_t^b = (1 - \phi_b) R_t^f$$

$$\left(1 + R_t^b\right) S_t = \beta (1 + R_t^f + \Phi_t + \Phi_{t-1}) S_{t+1}$$

### 2.1.4 Fiscal and Monetary Policy

The government takes in taxes from the households and engages in spending on traded goods. We assume that spending may be either pro-cyclical or counter-cyclical, depending on the value of $\rho_{GY}$, that there is smoothing in government consumption, and there is a stochastic component to spending:

$$G_t = (1 - \rho_g) G_{st} + \rho_g G_{t-1} + (1 - \rho_g) \rho_g (Y_t - Y_{st}) + \varepsilon_{g,t} \sim N(0, \sigma_{g,t}^2)$$

Tax revenues are sourced from labor and consumption and the fiscal borrowing requirement equals the familiar government budget balance:

$$TAX_t = \tau W_t L_t + \tau_c P_t C_t$$

$$B_t^f = (1 + R_{t-1}^b) B_{t-1}^f + P_t^b G_t - TAX_t$$
We assume that the central bank sets the policy rate $R_t^p$ according to the following Taylor-rule framework:

$$R_t^p = \rho_p R_{t-1}^p + (1 - \rho_p) R_{t}^p + (1 - \rho_p) \rho s \left( \frac{P_{t+1}}{P_t} - 1 - \pi_t \right) + \epsilon_{\text{p}_t}$$

where $R_{t}^s$ is the steady-state policy rate, $P_{t+1}/P_t = \pi_{t+1}$ is the expected inflation rate at time $t$ for time $t+1$, while $\pi_t$ is the policy-determined and announced target of inflation set by the central bank at time $t$.

We assume that the steady state policy rate, $R_{t}^s$, and the steady-state risk-free rate on domestic government bonds, $R_{t}^b$, are the same. However, we assume that the domestic bond market follows a stochastic process, in which it responds to its own lags as well as to differences between the policy rate and the lagged risk-free rate, as well as a white noise term. By adjusting the policy rate, the central bank, with a lag, induces changes in the bond rate, but it does so in a stochastic setting.

$$R_t^b = \rho_p R_{t-1}^b + (1 - \rho_p) R_{t}^b + (1 - \rho_p) \rho s (R_{t-1}^p - R_{t-1}^b) + \epsilon_{\text{b}_t}; \epsilon_{\text{b},t} \sim N(0, \sigma^2_{\epsilon_b})$$

Given that the central bank sets the policy rate, and thus affects the return on government bonds and the rates of return on deposits and loans, it is obliged to provide liquidity support to the banking system to sustain this policy. We assume a zero profit condition for the financial sector. Once the interest rates are in place as a result of policy decisions, the central bank provides liquidity support to the banking system to ensure balance-sheet balance:

$$LIQ_t = N_t + B_t + (1 + R_{t-1}^d - \phi_4 R_{t-1}^d) M_{t-1} + (1 + R_{t-1}^l + \Phi_4 R_{t-1}^l) B_t' S_t - (1 + R_{t-1}^l - \phi_5 R_{t-1}^l) N_{t-1} - M_t - (1 + R_{t-1}^b) B_{t-1} - B_t' S_t$$

The central bank can affect interest rates on loans and deposits by adjusting the required reserve ratio $\phi_4$ and the banking-sector capital asset ratio, $\phi_5$. These ratios can be allowed to vary, for example, when deposits or total loans are above their steady state values:

$$\phi_4 = \bar{\phi}_4 + \phi_4 (M_{t-1} - M_{ss})$$

$$\phi_5 = \bar{\phi}_5 + \phi_5 (N_{t-1} - N_{ss})$$
2.1.5 Foreign Debt and Interest Rates

The aggregate foreign borrowing evolves according to the absorption approach to the balance of payments. Current level of foreign debt is simply the sum of previous foreign indebtedness and current account balance.

\[ S_{t}B_{t}' = [1 + R_{t-1}' + \Phi_{t-1}']S_{t-1}B_{t-1}' + P_{t}^{m'}(C_{t}' + I_{t}') - P_{t}'(C_{t}') \]

The foreign interest rate, \( R_{t}' \), follows an autoregressive stochastic process:

\[ R_{t}' = \rho_{R}R_{t-1}' + (1 - \rho_{R}R)R_{ss} + \varepsilon_{R,t} \]

It should be noted that the risk premium embedded in the accumulation of foreign debt effectively closes this open economy model, so that the domestic consumption and foreign debt levels do not become indeterminate. There are other ways to close the open economy model, such as adjustment costs on foreign debt accumulation, or an endogenous discount factor. We feel that the incorporation of a time-varying endogenous risk premium is a more intuitive way to close this model.

3. Competitive Equilibrium in the Steady State

Before we can even think of solving the model and implementing it for policy simulation (much less Bayesian estimation) we have to find the steady state under the competitive equilibrium conditions. Why is that necessary?

First, the steady state is a good place to start for policy analysis. Starting anywhere else, the economy is in motion and it is hard to sort out what effects are due to the policy program and what effects are simply due to not being in the steady state.

Second, most of the efficient solution methods for these models involve first or even second-order (or even still higher) progressively more accurate Taylor expansions around the steady-states of the endogenous variables of the model. Without the steady states in hand, we are going nowhere with this or any other model.

The competitive equilibrium in the steady state, of course, implies that markets are clearing, and that variables which are either forward or backward-looking are at rest, so that, for example, \( C_{t} = C_{t+1} = C_{t+1} \) for all \( t \). For simplicity we label the steady-state representations of the endogenous variables in the same way as the variables in the dynamic model, but without a time subscript.

(a) Supply meets demand in exports and home goods production

\[ Y^{x} = C^{X} + C^{x*} \]

\[ Y^{h} = C^{h} + G \]
(b) Labor demand in each sector equals total labor supply

\[ L = L^h + L^x \]

\[ W = \frac{\gamma L^s}{\Lambda(1 - \tau)} \]

(c) There is labor mobility between industries such that real wage is the same

\[ \frac{\partial Y^h}{\partial L^h} \left( 1 + \phi_2 \right) R^f = \frac{\partial Y^x}{\partial L^x} \left( 1 + \phi_1 \right) R^f \]

(d) Government debt is constant so that tax revenues pay for spending and the interest rate on debt

\[ R^b B^g + P^h G = TAX \]

(e) Foreign debt is constant so that export revenues pay for imports and interest payments on foreign debt

\[ [R^f + \Phi]SB^f + P^f (C^f + I^x) = P^x C^x \]

(f) Investment is at its steady-state rate, \( I^x = \delta_i R^x \)

(g) Household’s steady state equilibrium conditions mean that revenue from labor, deposits, dividends and capital pay for consumption, tax, and steady-state investment.

\[ WL + R^d M + \Pi + P^k K^x = PC(1 + \tau_c) + \tau WL + P^m I^x \]

(h) For banks, return on government bonds and loans to firms pay for interest on foreign debt and deposits, opportunity cost of holding reserves against deposits and capital against assets

\[ R^b B^g + R^i N = (R^f + \Phi) B^f S + R^d M + \phi_4 M_{t-1} + \phi_5 N \]

(i) There are three steady-state conditions involving \( R^d \) and Tobin’s \( Q \) for saving and investment by households in the export industry

\[ \beta = \frac{1}{1 + R^d} ; \quad \Lambda = \frac{Q^x}{P^f} ; \quad \text{and} \quad Q^x = \left( \frac{1}{(1 - \beta(1 - \delta_i))} \right) \beta \Lambda P^k \]

Given the high degree of nonlinearity of the equations of the model, one would be hard put to find any analytical solution for the steady-state values of the model. Instead, we used a computational algorithm to solve for the steady state values, based on calibration of the structural parameters of the model. It should be clear that some structural parameters, such as the Taylor rule coefficients or the Calvo pricing parameters, do not affect the steady state solutions of the model. But other coefficients such as the relative risk aversion coefficient in the utility function, the production function
coefficients, the demand elasticities, and the tax rates do affect the steady state.

3.1 Calibration and Steady-State Solution

For many working in macroeconomics, calibration is the only way to approach empirical dynamic macroeconomic models. This approach experiments with different sets of parameters and attempts to find the best set of parameters by matching moments of the key endogenous variables of the model with corresponding moments of observed real world data. One popular method is to find the impulse response functions (IRFs) of the dynamic model calibrated with specific parameter values, and then find the impulse response functions of a vector autoregressive model estimated with actual observed data. The "goodness of fit" of the model is assessed by how well the impulse response functions of generated by the calibrated model match those generated by the VAR model estimated with real world data.

Using a VAR approach with impulse responses has been criticized as a poor way to assess the quality of fit of a calibrated nonlinear dynamic model. One obvious criticism is that the VAR specification is a poor approximation to the richer dynamic model. A second problem, of course, is that the VAR estimated with observed data is only one realization with a finite sample. There is no reason to expect that the impulse response estimated with one finite data set should converge to the underlying true impulse response.

3.2 Data

Data for the period 2002-2009Q3 were used. The observable variables consist of the cyclical component of the following deseasonalized series: private consumption, government consumption, investment, gross domestic product, exports, imports, and terms of trade. Nominal variables such as deposits and loans to the private sector, inflation rate, 90-day LIBOR, inflation target, BSP’s policy rate (reverse repurchase rate), weighted average lending rate of commercial banks, weighted average deposit rate of commercial banks, and change in exchange rate were simply detrended. All interest rate variables are expressed in gross terms.

3.3 Calibrated Parameters

Some parameters affect the steady state, others do not. The parameters which do not affect the steady state are easy to estimate when we turn to Bayesian estimation, since there is no need to solve for a new steady state when we alter the values of these parameters. However, for those parameters which affect the steady state, Bayesian estimation is very time consuming. Hence it is better to calibrate these parameters affecting the steady state and keep them fixed when we do Bayesian estimation.

3.3.1 Parameters Affecting the Steady State

The calibrated parameters appear in Table 1. Some of the specifications are without controversy. The discount factor $\beta$ is set at 0.99 for quarterly data. The relative risk aversion coefficient, $\sigma$ is set at 3.0. In a purely log linear consumption function, this parameter would be set at unity. Most studies of emerging market countries set this parameters at values greater than 1.5. The share of foreign-produced consumption goods in the overall consumption basket, $\lambda_1$ is set at 0.3, while the share of domestically produced goods in the basket of overall domestically produced consumption goods $\lambda_2$ is similarly set at 0.3. The habit persistence coefficient $h$ is 0.5. While this value is usually higher for studies in the United States, we assume that many consumers in emerging
market countries are less habitual. The labor supply elasticity, \( \varpi \), is 0.25 similar to that of other studies, while the disutility of labor \( \gamma \) is set at unity.

We assume that there is a higher intra-temporal substitution between domestic goods and foreign goods, than among differentiated home goods. The tax parameters for labor income and consumption, \( \tau \) and \( \tau_c \), are assumed to be equal, and set at 0.2. The CES coefficient for government spending in utility, \( \eta \), is set at 0.15, following a study by Pescharon for Mexico and Norway, while \( \kappa \) the CES coefficient for production, is set at -0.1, implying that the elasticity of substitution between capital and labor in the export production function is less than unity.

### 3.3.2 Parameters Not Affecting Steady State

The Taylor rule coefficients for the lagged interest rate and for the difference between expected inflation and target inflation, given by \( \rho^r \), \( \rho^\pi \) are at 0.9 and 1.5. The Taylor principle for determinacy of the price level requires that \( \rho^\pi \) should be greater than unity.

We assume that government spending is pro-cyclical, thus \( \rho_{gy} = 0.25 \). This has been documented in several studies of emerging market economies. The Calvo persistence parameter, \( \xi \), is typically set at 0.75 for quarterly data, implying a probability (above 0.10) of persistent prices for two years.

The standard deviations of the shocks and the autoregressive parameters are initially set at 0.1 and 0.9, respectively, implying a high degree of persistence and of course a large volatility for each shock. Bayesian estimation was applied to some parameters not affecting the steady state. In this manner, we will be able to determine the relative degree of persistence and the relative importance of each shock.

### Table 1. Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definitions</th>
<th>Calibrated Values</th>
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</thead>
<tbody>
<tr>
<td>( \rho_h )</td>
<td>Habit persistence</td>
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<tr>
<td>( \beta )</td>
<td>Discount factor</td>
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<td>( \lambda_1 )</td>
<td>Share of foreign consumption in consumption index</td>
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<tr>
<td>( \lambda_2 )</td>
<td>Share of export consumption in domestic consumption index</td>
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</tr>
<tr>
<td>( \sigma )</td>
<td>Relative risk aversion</td>
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<td>( \mu )</td>
<td>Coefficient of government consumption in the overall consumption index</td>
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</tr>
<tr>
<td>( \gamma )</td>
<td>Disutility of labor</td>
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<tr>
<td>( \varpi )</td>
<td>Labor supply elasticity</td>
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<td>Intra-temporal elasticity of substitution between domestic and imported consumption</td>
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<td>( \theta_2 )</td>
<td>Intra-temporal elasticity of substitution between domestically produced and export consumption</td>
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<tr>
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<td>CES utility coefficient</td>
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<td>( \kappa )</td>
<td>CES substitution parameter in production</td>
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<td>( \phi_f )</td>
<td>Risk premium parameter</td>
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<tr>
<td>( \phi_2 )</td>
<td>Borrowing requirements of home goods producers</td>
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<tr>
<td>( \phi_3 )</td>
<td>Borrowing requirements of importer</td>
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<tr>
<td>( \phi_4 )</td>
<td>Required reserves of deposits</td>
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</tr>
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</table>
### 3.3.3 Solution of the Steady State

The solution of the steady state for the endogenous variables must respect the competitive equilibrium conditions as well as the inter-temporal budget constraints under the condition that for each dynamic variable \( x_t = x_{t-1} = x_{t+1} \) for all \( t \).

This means that the following relationships must hold for Tobin’s \( Q \), the steady state marginal utility of wealth \( \Lambda \), and the marginal productivity of capital in the export sector, \( P^k \):

(a) \( Q^* = \left( \frac{1}{a - \beta (1 - \delta)} \right) \beta \gamma P^k = P^f \Lambda \)

This relation implies: \( P^f = \left( \frac{1}{a - \beta (1 - \delta)} \right) \beta \gamma P^k \)

where:

\[
\Lambda = \left[ (\bar{C})^{1-\rho} \right]^{\frac{1}{\gamma}} \left( \frac{1}{H} \right) \left( \tilde{C}^{1-\eta} \phi(C)^{-1} \right) \\
\bar{C} = \left[ \phi C^{-\eta} + (1 - \phi) G^{-\eta} \right]^{-\frac{1}{\eta}} \\
P = \left[ (1 - \lambda) \left( P^d \right)^{1-\theta} + \lambda_e \left( P^f \right)^{1-\theta} \right]^{-\frac{1}{\theta}}
\]

(b) The current account is in equilibrium:

\[
[R^f + \Phi SB^f + P^f (C^f + I^*)] = P^s C^s
\]
(c) The government budget is balanced:
\[ R^b B^g + P^b G = TAX \]

(d) The household budget is balanced:
\[ WL + R^d M + \Pi + P^x K^x = PC(1 + \tau_c) + \tau WL + P^f I^x \]

(e) The banking sector does not need liquidity injections from the central bank:
\[ R^b B^g + R^l N = (R^f + \Phi)B^f S + R^d M + \phi_4 M_{t-1} + \phi_5 N \]

(f) The following conditions for labor hold:
\[ W = \frac{\gamma (L^h + L')}{\Lambda (1 - \tau)} \]
\[ \frac{\partial Y^h}{\partial L^h} \frac{P^h}{(1 + \phi_2)R^f} = \frac{\partial Y^x}{\partial L^x} \frac{P^x}{(1 + \phi_1)R^f} \]

(g) Finally, output demand in each sector is equal to output supply
\[ Z^h A^h [(1 - \alpha_i)(L^i)^{-\kappa_i} + \alpha_i (K^i)^{-\kappa_i}]^{\frac{1}{\kappa_i}} = C^f + C^x \]
\[ Z^h (L^h)^{1-\alpha_2} = C^h + G \]

We thus have nine nonlinear equations. This means that we can solve for nine steady-state endogenous variables. We choose to solve for the steady-state values of \( C, C^*, G, W, L^h, L^l, K^x \) and \( M \). For simplicity we fixed the steady state value of the exchange rate at unity, \( S=1 \). We have one more variable required to ensure a steady-state solution. In many models, the coefficient of disutility of labor, \( \gamma \), is calibrated as a steady-state solution. We found that it was quicker to set this parameter to unity, and solve for the constant term in the production function for exports, \( A^x \). Once we have these variables pinned down, we can solve for the price indices and values of consumption in home and foreign goods.

This system is a non-recursive, nonlinear system. Fortunately there are numerical methods for solving such systems. Matlab has a command called FSOLVE, a nonlinear equation solver, in which we set up the nine equations equal to zero. Thus:

\[ 0 = P^f - \left( \frac{1}{a - \beta (1 - \delta_i)} \right) \beta A P^k \]
\[ 0 = [R^f + \Phi]SB^f + P^f (C^f + I^x) - P^x C^x \]
\[ 0 = R^h B^g + P^h G - TAX \]
\[ 0 = WL + R^d M + \Pi + P^x K^x - [PC(1 + \tau_c) + \tau WL + P^f I^x] \]
\[ 0 = R^h B^g + R^l N - (R^f + \Phi)B^f S - [R^d M + \phi_4 M_{t-1} + \phi_5 N] \]
To solve this system, we want to minimize the sum of the squared errors of the left hand side of the equations, for the choice of steady state values of $C, C^\ast, G, W, L^h, L^k, K^x, M, A^x$

$$\text{Min} \quad \sum_{i=1}^{9} \varepsilon_i^2$$

subject to the following constraints:

$$\varepsilon_1 = P^f - \left( \frac{1}{a - \beta(1 - \delta)} \right) \beta \Lambda P^k$$

$$\varepsilon_2 = [R^* + \Phi]SB^I + P^f (C^f + I^k) - P^a C^x$$

$$\varepsilon_3 = R^h B^k + P^h G - TAX$$

$$\varepsilon_4 = WL + R^d M + \Pi + P^{kx} K^x - [PC(1 + \tau_c) + \tau WL + P^f I^x]$$

$$\varepsilon_5 = R^h B^k + R^d N - (R^f + \Phi) B^f S - [R^d M + \phi_4 M_{t-1} + \phi_2 N]$$

$$\varepsilon_6 = W - \frac{\gamma(L^h + L^f)}{\Lambda(1 - \tau)}$$

$$\varepsilon_7 = \frac{\partial Y^h}{\partial L^h} \frac{P^h}{(1 + \phi_2 R^f)} - \frac{\partial Y^x}{\partial L^x} \frac{P_x}{(1 + \phi_1 R^f)}$$

$$\varepsilon_8 = Z^x A^I \left[ (1 - \alpha_1)(L^x)^{-\kappa_1} + \alpha_1(K^x)^{-\kappa_1} \right]^{\frac{1}{\kappa_1}} - [C^f + C^*]$$

$$\varepsilon_9 = Z^h (L^h)^{1-\alpha_2} - [C^h + G]$$

and $C > 0, C^* > 0, G > 0, W > 0, L^h > 0, L^f > 0, K^x > 0, M > 0, A^x > 0$

However, non-linear optimization is prone to the local minimum trap instead of global minimum, depending on the starting value. There are no absolute solutions to
local minimum problems but there are strategies involving re-estimation or 'stochastic evolutionary search. The approach used in the paper approach involves three phases: 

(a) Local 'gradient-based' search, which searches for the first and second order derivatives and continually updates the derivatives until some stopping criteria are reached; (b) The results from the gradient-based search serve as the initial conditions that are fed into another stochastic search method called simulated annealing. Just like the gradient-based approach, simulated annealing is also not strictly a global search. Rather, it is a random search method that helps move to a better minimum; 

and (c) The results from simulated annealing then serve as the initial guesses for the stochastic evolutionary search called 'genetic algorithm. The population of initial guesses are updated by genetic selection and mutation for many generations until the 'best' among the last generation of populations.

4. Policy Simulations

Two policy simulations that were of interest during the crisis period were carried out using the model: reduction in policy rate and fiscal stimulus via higher expenditures. In policy simulations, one variable changes for a given period of time, by a pre-specified amount, while the other exogenous variables are kept at their steady state values.

4.1 Interest Rate Policy

The simulations consider a temporary (annualized) 25 bps reduction in BSP’s reverse repurchase (RRP) rate for a period of 8 quarters. The policy shock results in higher price level and hence, inflation. Domestic absorption rises, putting upward pressure on prices of non-tradeables. As cost of credit goes down, credit likewise expands. Lower interest rate also leads to nominal exchange rate depreciation.

Lower cost of credit and nominal exchange rate depreciation initially encourage production in the tradeable sector, bringing about a small initial increase in output. The initial expansion in tradeable output eventually raises the demand for labor in the trade sector, putting pressure on aggregate nominal wage and hence, on marginal cost. The resulting higher price expectation pushes up the growth of the aggregate price level.

The higher price level eventually outweighs the nominal exchange rate depreciation, causing real exchange rate appreciation. As a result, there is a much larger contraction in later periods as output in the tradeable sector dips due to real exchange rate appreciation.

3 The term 'annealing' derives from metallurgy which entails heating and controlled cooling of a material to increase the size of the crystals and reduce the defects. Heat causes the atom to become unfastened from initial conditions (or the local minimum of the internal energy) and move randomly to states of higher energy. The slow cooling allows greater chances of finding combinations with lower internal energy than the initial one.
4.2 Fiscal Policy Shock

A fiscal stimulus equivalent to one percent of GDP leads to initial rise in domestic absorption (government consumption) and correspondingly output. Since government spending is largely a non-traded good, the ensuing expansion in non-tradeable output also leads to higher demand for labor and thus, nominal wages. Higher nominal wage feeds into marginal cost, which eventually pushes up prices of non-tradeables and feeds into overall inflation. With higher nominal wage, firms (both non-traded firms and exporters) borrow a greater portion of their working capital for their wage bill, resulting in higher lending.

Higher inflation sets off higher interest rate and causes nominal exchange rate appreciation. The nominal exchange rate appreciation and higher price of non-tradeables leads to real exchange rate appreciation, reducing exports and resulting in lower current account (CA) balance. The decline in CA balance, in turn, translates into accumulation of foreign debt and higher risk premium. The higher risk premium fuels nominal exchange rate depreciation. Eventually, this dampens output and inflation.

The resulting fiscal multiplier is somewhat low. This result is consistent with the Mundell-Fleming and Dornbusch models, which show that fiscal policy is muted in an economy with flexible exchange rates and capital mobility.
5. Conclusion and Areas for Further Research

The BSP’s DSGE model is a small open economy model with habit persistence, staggered pricing in home goods production, flexible wage, adjustment cost to investment, and financial frictions.

The two simulation results, however approximate, illustrate the inter-temporal trade-offs in each policy decision that policymakers need to take into account. The effects of policy rate reduction on inflation and output are more apparent in the tradeable sector whereas fiscal stimulus works via the non-tradeable sector. The low fiscal multiplier obtained from the simulation on fiscal stimulus is consistent with the Mundell-Fleming and Dornbusch models, which show that fiscal policy is muted in an economy with flexible exchange rates and capital mobility.

There are extensions and modifications that can be introduced. Further experiments with different parameter calibrations are interesting areas of future research to test the robustness of the results. The model, however, can only work with small shocks because the solution method used is still the log linearization method. As developments in estimation methods are continuously evolving, we can expect more powerful computational methods that would enable researchers to account for big shocks or crisis scenarios.
References


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